Hardened Concrete

Lecture No. 16
Fatigue strength of concrete

- Modulus of elasticity,
- Creep
- Shrinkage of concrete
Stress-Strain Plot of Concrete

- At stress below 30% of ultimate strength, the transition zone cracks remain stable. The stress-strain plot remains linear.

- At stress between 30% and 50% of ultimate strength, the transition zone microcracks begin to increase in length, width and numbers. The stress-strain plot becomes non-linear.

- At 50 to 60% of the ultimate stress, cracks begin to form in the matrix. With further increase to about 75% of the ultimate stress, the cracks in the transition become unstable, and crack propagation in the matrix will increase. The stress-strain curve bends towards the horizontal.
Stress-Strain Plot of Concrete

(4) At 75 to 80% of the ultimate stress, the stress reaches a critical stress level for spontaneous crack growth under a sustained stress. Cracks propagate rapidly in both the matrix and the transition zone. Failure occurs when the cracks join together and become continuous.

Concrete is not a truly elastic material, as evident from the nonlinear stress-strain curve for concrete, shown in the fig.
Stress-Strain Plot of Concrete

- The “initial tangent modulus” is given by the slope of a line drawn tangent to the stress-strain curve at the origin.
- The “tangent modulus” is given by the slope of a line drawn tangent to the stress-strain curve at any point on the curve.
- The “secant modulus” is given by the slope of a line drawn from the origin to a point on the curve corresponding to a 40% stress of the failure stress.
- The “secant modulus” is given by the slope of a line drawn from the origin to a point on the curve corresponding to a 40% stress of the failure stress.
Modulus of elasticity for concrete determined from an experimental stress-strain relation curve, as described above, is generally termed as static modulus of elasticity ($E_c$) whereas the modulus of elasticity determined through the longitudinal vibration test is termed as dynamic modulus of elasticity ($E_d$).
Determination of modulus of elasticity of Concrete

- Testing of cube or cylinder in uni-axial compression test.
- Measure load and the corresponding deformation as the load is increased. Draw the stress strain curve.
- Strain = Dial gauge reading/gauge length = $dl/L$
- Stress = Load/Cross sectional area = $P/A$
- Use Compressometer and Extensometer to measure deformations. Draw stress strain diagram and determine the required modulus.
- Deflection: $E$ can be determined from testing of beam also.
Determination of modulus of elasticity

- For central point load, Max. deflection,

\[ \delta = \frac{Wl^3}{48EI_{xx}} \]
Determination of modulus of elasticity

- The test uses a 150 × 300 mm cylindrical specimen, which is loaded in compression. A compressometer is used to measure the longitudinal strains, and an extensometer is used to measure the transverse strains on the specimen.

- The chord modulus (E) is calculated as:

\[ E = \frac{(S_2 - S_1)}{(\varepsilon_2 - 0.00005)} \]

- where \( S_2 \) = stress corresponding to 40% of ultimate strength
- \( S_1 \) = stress corresponding to a strain of 50 × 10^{-6}
- \( E_2 \) = longitudinal strain produced by stress \( S_2 \)
Poisson’s ratio (Static Method)

- When a material is compressed in one direction, it usually tends to expand in the other two directions perpendicular to the direction of compression. This phenomenon is called the Poisson effect.

- The Poisson ratio is the ratio of the fraction (or percent) of expansion divided by the fraction (or percent) of compression, for small values of these changes. $\mu = 0.15 - 0.20$ – Actual value to be found from strain measurements on concrete cylinder using extensometer.

$$
\mu = \frac{(\varepsilon_{t2} - \varepsilon_{t1})}{(\varepsilon_2 - 0.00005)}
$$

where $\varepsilon_{t2}, \varepsilon_{t1}$ = transverse strains produced by $S_2$ & $S_1$, respectively.
Poisson’s ratio (Static Method)

- An alternate method for finding Poisson’s ratio is from UPV test and by finding the fundamental natural frequency of longitudinal vibration of concrete beam. The Poisson’s ratio can be found from the following equation.

- The Poisson’s ratio is slightly higher and it ranges from 0.2 to 0.24.

\[
\left[ \frac{V^2}{2nL} \right]^2 = \frac{1 - \mu}{(1 + \mu)(1 - 2\mu)}
\]

\[
E_d = \rho V^2 \frac{(1 + \mu)(1 - 2\mu)}{(1 - \mu)}
\]
Poisson’s ratio (Static Method)

- Where $V =$ Pulse velocity mm/s
- $n =$ Resonant frequency in Hz and
- $L =$ Length of the beam in mm.
- $\rho =$ The density of concrete
- $Ed =$ Dynamic modulus of elasticity of concrete
Modulus of elasticity of concrete increases approximately with the square root of the strength. The IS 456 of 2000 gives the Modulus of elasticity as

$$E_c = 5000 \sqrt{f_{ck}}$$

<table>
<thead>
<tr>
<th>Average compressive strength of works cubes MPa</th>
<th>Modulus of Elasticity GPa</th>
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<tbody>
<tr>
<td>21</td>
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<tr>
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<td>42.9</td>
</tr>
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<td>70</td>
<td>46.4</td>
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Factors Affecting Modulus of Elasticity of Concrete

- Effects of moisture condition
  - Specimens tested in dry condition show about 15% decrease in elastic modulus as compared to the wet specimens. This is explained by the fact that drying produces more microcracks in the transition zone, which affects the stress-strain behavior of the concrete.
  - This is opposite to its effects on compressive strength. The compressive strength is increased by about 15% when tested dry as compared with the wet specimens.
Factors Affecting Modulus of Elasticity of Concrete

- Effects of Aggregate properties

Porosity of aggregate has the most effect on the elastic modulus of concrete. An aggregates with a low porosity has a high modulus of elasticity.

The elastic modulus of concrete is affected by the volume fraction of the aggregate as well as the elastic modulus of the aggregate.
Factors Affecting Modulus of Elasticity of Concrete

- Effects of cement matrix
  - The lower the porosity of the cement paste, the higher the elastic modulus of the cement paste.
  - The higher the elastic modulus of the cement paste, the higher the elastic modulus of the concrete.

- Effects of transition zone
  - The void spaces and microcracks in the transition play a major role in affecting the stress-strain behavior of concrete.
  - The transition zone characteristics affect the elastic modulus more than it affects the compressive strength of concrete.